

**Carderock Division  
Naval Surface Warfare Center  
West Bethesda, MD 20817-5700**

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**CARDIVNSWC-TR-61-99-20 August 1999**

Survivability, Structures, and Materials Directorate  
Technical Report

**Development of Compliance Expression for  
Deeply Flawed 1T-WOL Specimen**

by

P.W. Holsberg

R.A. Hays

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## **ABBREVIATIONS**

a	= Crack Length
$a_0$	= Initial Crack Length
$a_f$	= Final Crack Length
B	= Net Specimen Depth ( $\sqrt{B}B_N$ )
$C_1$	= Distance from Load Line to Clip Gage Location (0.65 in.)
E	= Young's Modulus ( $29 \times 10^6$ psi)
$G_I$	= Strain Energy Release Rate
$K_I$	= Stress Intensity Factor
P	= Load
$U_{tot}$	= Total Strain Energy
$U_{no\ crack}$	= Strain Energy with No Crack
$U_{crack}$	= Strain Energy with Crack
$V_B$	= Crack Opening at Load Line
$V_G$	= Crack Opening at Clip Gage Location
W	= Specimen Width (2.55 in.)
$\nu$	= Poisson's Ratio (0.3)

## **ABSTRACT**

It is difficult to make precise load measurements in 1T-WOL specimens where significant crack extension (the ratio of the final crack length to specimen width exceeds 0.8) has occurred during testing. An alternate method to determine this load would be from compliance calculation using the original, measured crack mouth opening displacement. Such an equation has been derived and verified.

## **ADMINISTRATIVE INFORMATION**

The Metals Department (Code 61) of the Naval Surface Warfare Center, Carderock Division (NSWCCD) performed this task. The work was sponsored by Dr. John Sedriks, Office of Naval Research, ONR Code 332.

## **INTRODUCTION**

The modified Wedge Open Load (WOL) test [Novack and Rolfe] utilizes a specimen which is convenient for the determination of the susceptibility of a high strength steel, weldment, or other alloy to stress corrosion or other environmentally assisted cracking. When conducting tests on various product forms (plate, castings, weldments, etc.) in tough, high strength steels, high initial loads are desirable to minimize incubation time. For example, a crack tip stress intensity of 120 ksi/in. in a 1-inch thick WOL specimen can be achieved with a load of approximately 25,000 lb and a corresponding crack mouth opening of about 0.03 in. If the steel product under test turns out to be very susceptible to cracking during that test, the crack may extend deeply into the specimen, reducing the residual load to the order of hundreds of pounds. The work done by Novack and Rolfe allows calculations of stress intensity and compliance of the WOL specimen to 80% crack through the test ligament,  $a/W \leq 0.8$ , Figure 1. A formula developed by Wilson allows calculation of stress intensity where  $a/W > 0.8$ , but no compliance formula was derived. Problems with corrosion and deposits on the crack surface, together with an uneven crack front profile, can make precise measurement of the after-test loads in the specimen difficult. The after-test loads are usually determined in the same fixtures and machines used to pre-load the specimen, also affecting the precision of such measurements. It would be useful, therefore, to be able to calculate the remaining load.

The objective of this study was to develop and verify an equation for calculating the remaining load by compliance utilizing the initial crack mouth opening.

## ANALYSIS

An expression relating compliance of deeply cracked WOL's to specimen geometry and material properties can be obtained by using the methods [Bucci et al.], and the  $K_I$  expressions [(Novak and Rolfe), (Wilson)] described in the literature.

The total amount of strain energy stored in a WOL specimen, Figure 1, is\*:

$$U_{tot} = U_{no\ crack} + U_{crack} \quad (1)$$

For a WOL specimen, the strain energy contribution of the uncracked specimen is negligible,  $U_{no\ crack} = 0$ . Therefore,

$$U_{tot} = U_{crack}. \quad (2)$$

To determine the strain energy due to the crack, using the Griffith approach [Anderson]:

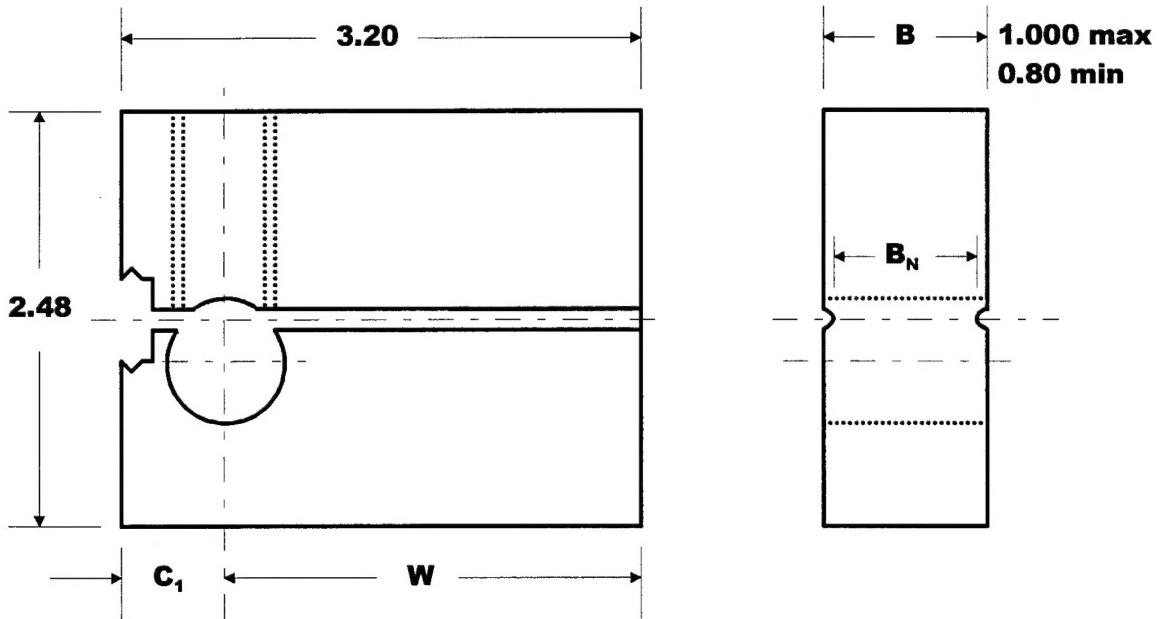
$$G = \frac{\partial U_{crack}}{B \partial a},$$

from which:

$$U_{crack} = B \int_0^a G \partial a.$$

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\* See List of Abbreviations for explanation of terms.



**Figure 1. 1T WOL Specimen**

Under plane-strain conditions:

$$K_I^2 = \frac{EG}{(1-\nu^2)}, \quad (3)$$

Or

$$G = \frac{(1-\nu^2)}{E} K_I^2. \quad (4)$$

Substituting equation (4) in (2):

$$U_{crack} = \frac{B(1-\nu^2)}{E} \int_0^a K_I^2 da. \quad (5)$$

Substituting equation (5) in (1):

$$U_{tot} = \frac{B(1-\nu^2)}{E} \int_0^a K_I^2 da .$$

We are interested in deeply cracked WOL specimens where  $a/W$  is greater than 0.8. Since there is no expression for  $K_I$  which covers the entire range, we must perform the operation stepwise. Thus:

$$U_{tot} = U_{a/W < 0.8} + U_{a/W > 0.8} \quad (6)$$

For  $a/W < 0.8$ , the expression for  $K_I$  is [Novak and Rolfe]:

$$K_I = \frac{PC_3(a/W)}{Ba^{\frac{1}{2}}} \quad (7)$$

where

$$C_3(a/W) = 30.96(a/W) - 195.8(a/W)^2 + 730.6(a/W)^3 - 1189.3(a/W)^4 + 754.6(a/W)^5$$

Substituting equation (7) in (5) and performing the integration from  $a/W = 0$  to  $a/W = 0.8$  we find:

$$U_{a/W < 0.8} = \frac{(1-\nu^2)P^2}{BE} (157) . \quad (8)$$

For  $a/W > 0.8$ , the expression for  $K_I$  is [Wilson]:

$$K_I = \frac{P(5W+3a)}{2B(W-a)^{\frac{3}{2}}} \quad (9)$$

Substituting equation (9) in (5) and performing the integration from  $a = 2.04$  ( $a/W = 0.8$ ) to  $a$  we find:

$$U_{a/W>0.8} = \frac{(1-\nu^2)P^2}{4BE} \left[ -587 + \frac{(5+3(a/W))^2}{2(1-(a/W))^2} - 3 \frac{(5+3(a/W))}{(1-(a/W))} - 9 \log(1-(a/W)) \right]. \quad (10)$$

Combining equations (8) and (10) with (6):

$$U_{tot} = \frac{(1-\nu^2)P^2}{4BE} \left[ 41 + \frac{(5+3(a/W))^2}{2(1-(a/W))^2} - 3 \frac{(5+3(a/W))}{(1-(a/W))} - 9 \log(1-(a/W)) \right] \quad (11)$$

Use of Castigliano's theorem [Popov] gives the load point crack opening displacement as:

$$V_B = \frac{\partial U_{tot}}{\partial P}$$

Applying this to equation (11)

$$\frac{BV_B}{P} = \frac{(1-\nu^2)}{2BE} \left[ 41 + \frac{(5+3(a/W))^2}{2(1-(a/W))^2} - 3 \frac{(5+3(a/W))}{(1-(a/W))} - 9 \log(1-(a/W)) \right] \quad (12)$$

Equation (12) relates the load point crack opening,  $V_B$ , to the applied load,  $P$ . However, most experimental compliance measurements done on WOL specimens use a NASA type clip gage which measures the crack opening not at the bolt but at the edge of the specimen. Novak and Rolfe interpolated from the gage displacement to the bolt displacement using the linear relationship:

$$V_B = \frac{aV_G}{a + C_1} \quad (13)$$

Experiments were performed on a WOL specimen where crack opening at the bolt was measured using a linear variable differential transformer mounted on the side of the specimen. A plot of  $V_G$  versus  $V_B$  was made during bolt loading. The results are summarized in Table 1.

**Table 1. Bolt-Loaded 1T WOL Specimen**

Crack Length $a$ , (in.)	$a/W$	Crack Opening		$V_B/V_G$	$a/(a + C_1)$
		$V_G$ (mils)	$V_B$ (mils)		
0.930	0.368	25	14.8	0.592	0.589
1.305	0.484	25	16.2	0.648	0.668
1.674	0.662	30	22.5	0.750	0.720
2.033	0.804	35	26.8	0.766	0.757
2.282	0.902	35	27.0	0.771	0.778

Some curvature of the plots was apparent in the first three tests, with  $V_B/V_G$  ratios as high as 0.7 being observed for the first 0.005 in. of deflection at 0.930 in. crack length. The final two tests, which are of particular interest here, were linear. Therefore, the linear correction, equation (13), can be applied to equation (12):

$$\frac{BV_G}{P} = \frac{(1-\nu^2) \left( \frac{a+C_1}{a} \right)}{2E} \left[ 41 + \frac{(5+3(a/W))^2}{2(1-(a/W))^2} - 3 \frac{(5+3(a/W))}{(1-(a/W))} - 9 \log(1-(a/W)) \right]. \quad (14)$$

A number of specimens were deeply sawcut and the compliance measured experimentally. Figure 2 shows the results of these tests, together with the values calculated from equation (14) and an empirical formula, equation (15) [Novak and Rolfe]. The calculated values are in excellent agreement, both being slightly higher than the experimental values. The agreement is felt to be satisfactory.

$$P = \frac{EBV_G}{C_6(a/W)} \quad (15)$$

where:

$$C_6(a/W) = \exp[3.453 - 8.097(a/W) + 42.314(a/W)^2 - 64.667(a/W)^3 + 36.845(a/W)^4].$$

Equation (14) must now be applied to a bolt-loaded specimen. Novak and Rolfe assumed a rigid bolt in his analysis and the same will be done here as the error is less than 10% and is conservative. This means that  $V_B$  is constant throughout the test. Examination of equation (13) shows, therefore, as the crack extends (i.e., as  $a$  increases) that  $V_G$  must decrease to maintain  $V_B$  constant. However,  $V_G$  is measured for all specimens on initial loading, so it is convenient to express equation (13) as follows:

$$V_B = \frac{a V_{G0}}{a_o + C_1} \quad (16)$$

where:

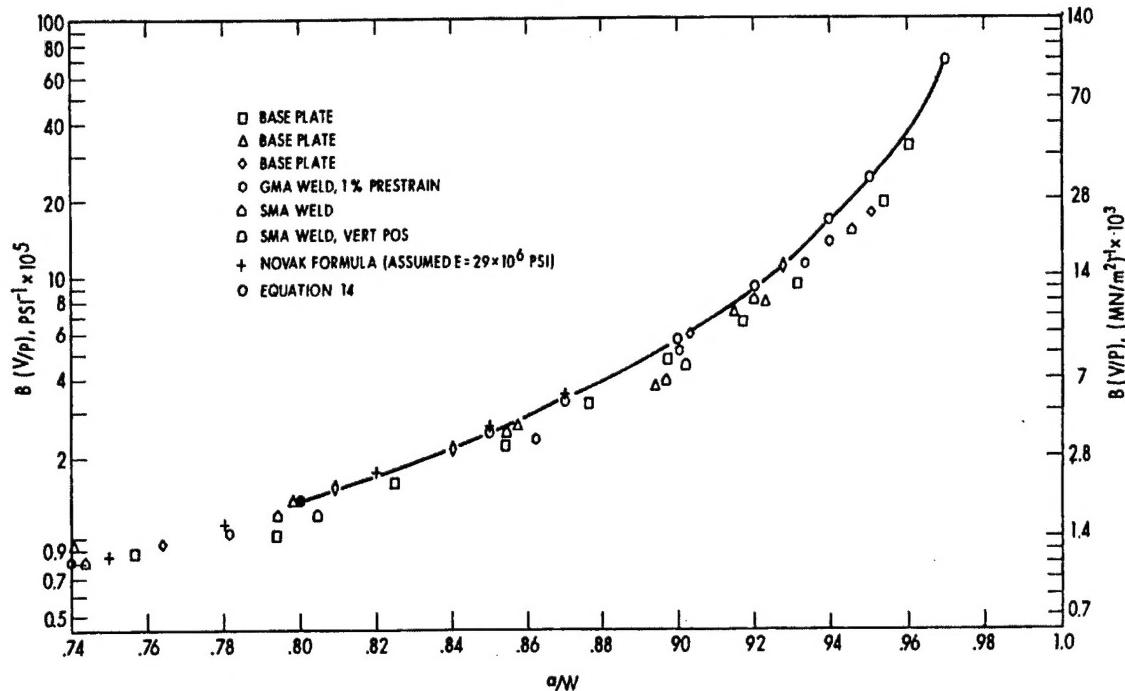
$a_o$  = initial crack length

$V_{G0}$  = initial crack opening at gage location.

Substituting equation (16), rearranging terms, and using  $a_f$  to indicate the crack length at the end of the stress-corrosion test, we find the expression for the remaining load to be:

$$P = \frac{2BEV_{G0} \left( \frac{a_o}{a_o + C_1} \right)}{(1-\nu^2) \left[ 41 + \frac{(5+3(a_f/W))^2}{2(1-(a_f/W))^2} - 3 \frac{(5+3(a_f/W))}{(1-(a_f/W))} - 9 \log(1-(a_f/W)) \right]} \quad (17)$$

Equation (17) can be used to calculate loads in bolt loaded specimens where the final  $a/W > 0.8$ .



**Figure 2. Compliance Curve for 1T-WOL Specimens**

## CONCLUSION

An expression, equation (17), has been derived relating the load remaining in a deeply flawed 1T-WOL specimen to the initial crack mouth opening displacement. This compliance relationship was verified by actual testing.

## ACKNOWLEDGEMENTS

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